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SMRI Spring 2009 Technical Conference
27-28 April 2009
Krakow, Poland

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SUMMARY

A brine-outflow test was performed in a 250-m deep cavern of the Gellenoncourt brine field. The cavern is opened, and the flow of brine expelled from the cavern is measured. The average brine-outflow rate is 12 liters/day. It is believed that this outflow rate mostly is due to cavern-creep closure, which appears to be 1.8×10^{-5}/yr, a figure significantly faster than expected for a shallow cavern. In fact, the observed outflow rate varies considerably with time, as it is influenced dramatically by atmospheric pressure fluctuations and, to a lesser extent, by Earth tides.

1. INTRODUCTION

CSME has operated a brine field at Gellenoncourt in Eastern France since the beginning of the 20th Century; this brine field has been described by Buffet (1998). Figure 1 presents a map of the brine field. It is located at the eastern (and shallowest) edge of the Keuper bedded-salt formation of Lorraine-Champagne, in which the salt thickness is 150 m. Five horizontal "salt pencils" have been described by geologists. The salt content of this field is highest in the first (shallowest) pencil and in the third pencil. The overburden layers include argillite, dolomite, sandstone and limestone.

During the first half of the 20th Century, single wells were brined out. After 1965, the hydrofracturing technique was used. For this brine field, cased and cemented wells are drilled to a depth of 280-300 m — i.e., at the base of the third pencil. The horizontal distance between two neighboring wells typically is 100 to 150 m. A link is created between the two caverns at the base of the third pencil through hydrofracturing. Water then is injected in one well, and brine is withdrawn from the other well. Caverns grow and their roofs actually reach the first pencil. Brining stops when the cavern roof is 10 m below the salt roof. This 10-m-thick salt slab is left to protect the overlying strata, which are prone to weathering when in contact with brine (Buffet, 1998).

In 2007, CSME decided to perform several field tests to prepare an abandonment report (Field abandonment is not considered at this time, but the regulatory authorities require that abandonment procedures be defined at an early stage.) The SG13-SG14 cavern was selected for performing in-situ tests, as this cavern is representative of the field and has been kept idle for a long period of time.

The SG13 and SG14 wells were drilled in May 1975, and operated as brine-production caverns from July 1976 to June 1977 (SG13), and from October 1978 to July 1980 (SG14). After some time, the two caverns coalesced, and SG13-SG14 now is composed of two parts connected by a large link; hydraulically, they can be considered as a single cavern. From latest sonar measurements (2000), it is inferred that the volumes of SG13 and SG14 are 107,000 m^3 and 34,000 m^3, respectively. However, sonar measurements are likely to underestimate the overall cavern volume, as they cannot "see" the insoluble-filled link between the two caverns. The vertical cross- section of the caverns is provided in Figure 2; a 3D view is provided in Figure 3.

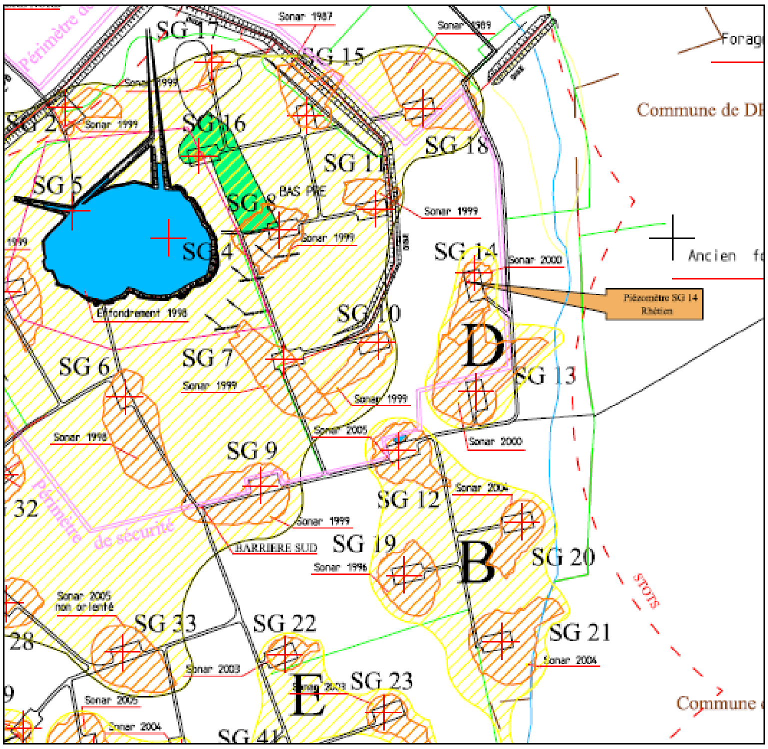

Figure 1 – Location of the SG13-SG14 cavern.

[The SG4-SG5 collapse was described by Buffet (1998).]

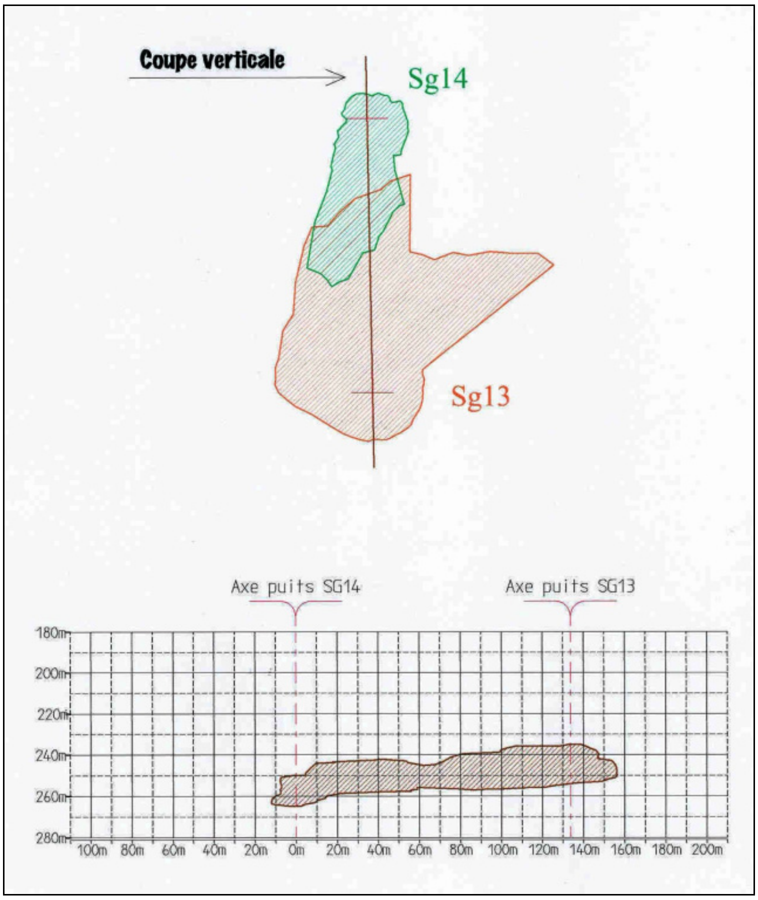

Figure 2 – Horizontal cross-section.

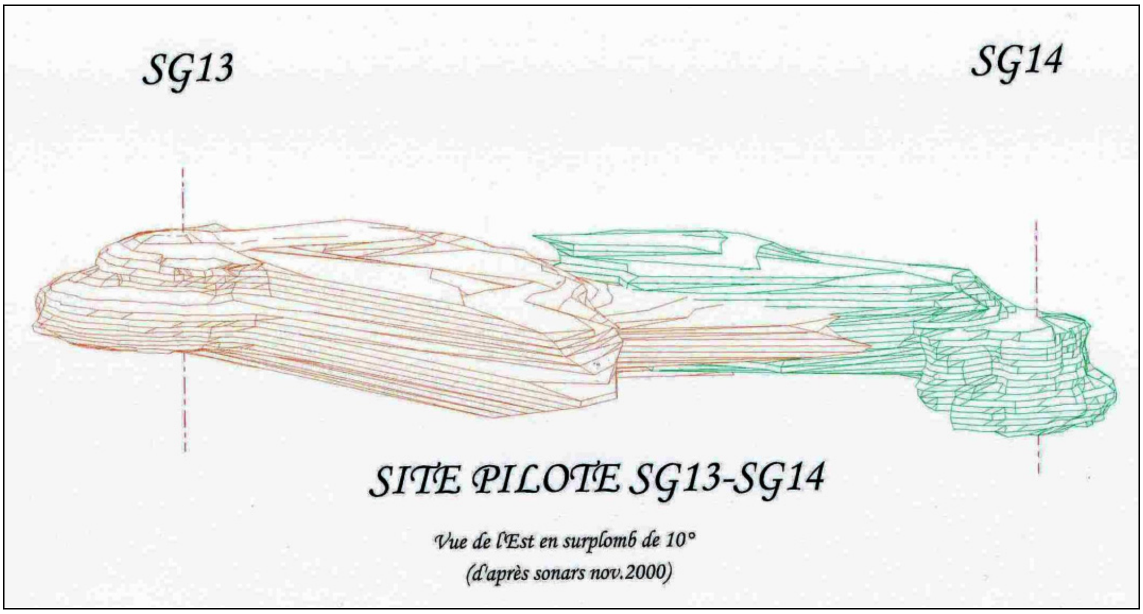

Figure 3 – 3D view of the SG13-SG14 cavern.

2. BRINE-OUTFLOW TESTS

When cavern abandonment is considered, the rate of cavern creep closure must be assessed precisely. In-situ tests are especially important in this context. Shut-in pressure tests and expelled flow-rate tests can be performed as well.

Shut-in pressure tests consist of closing the cavern and measuring the pressure evolution at the wellhead as a function of time. It is better to measure the pressure evolution both in the annular space and in the central tubing to provide redundancy and to assess possible casing leaks.

Liquid out-flow tests consist of opening the cavern and measuring the flow of liquid (brine or hydrocarbon) expelled from the wellhead (Figure 4). For example, the expelled volume can be collected in a container, and the fluid volume or container weight can be measured daily. In a small cavern, the daily flow rate is relatively small, and measurements can be taken automatically. One important asset of liquid-outflow tests is that, when properly assessed, as explained below, the observed flow rate certainly is faster than the long-term brine flow rate from the cavern to the overlying strata.

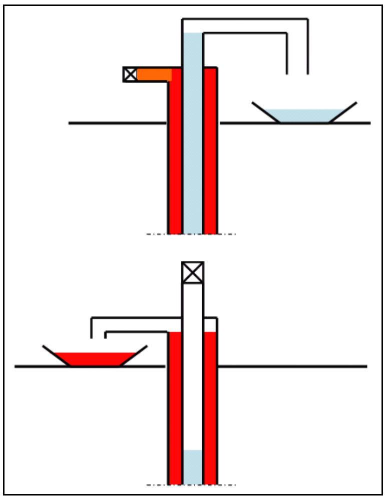

Figure 4 - Brine-outflow (top) and hydrocarbon-outflow (bottom) tests.

Outflow tests have been described in the literature; see, for example, Clerc-Renaud and Dubois (1980), Hugout (1988) and Brouard et al. (2004).

The liquid-outflow rate is governed by two main phenomena:

(1) cavern-creep closure rate; and

(2) cavern-brine thermal expansion.

In fact, when a brine-outflow test is performed in a cavern that has been kept idle for a long period of time, brine thermal expansion often can be disregarded.

Two other phenomena also may play a role:

(1) brine micro-permeation through the cavern walls; and

(2) brine leaks through the casing and casing shoe.

However, the significance of leaks and micro-permeation during an outflow test often is minor. During a brine-outflow test, the central tubing is open at ground level and is filled with saturated brine; consequently, the cavity pressure is halmostatic. It is well known from Mechanical Integrity Tests (Van Sambeek et al., 2005) that leaks and micro-permeation are large only when the cavern pressure is much larger than halmostatic.

Several more-or-less periodic phenomena also influence the brine-outflow rate:

- atmospheric pressure variations;
- ground-level temperature variations; and
- Earth tides.

However, when the testing period is sufficiently long (say, several weeks), the average effect of these periodic phenomena is nil, and the average brine-outflow rate mainly depends on the cavern-creep closure rate and the cavern-brine thermal expansion.

In a deep cavern, the cavern closure rate and the brine thermal-expansion rate are fast, and the other phenomena are not able to make the brine outflow vanish. Only small fluctuations of the brine flow rate can be observed; a precise interpretation of these fluctuations allows for assessment of the effects of Earth tides (Bérest et al., 1992). In a shallow cavern, the closure rate and the thermal expansion rate often are slow and the other phenomena play a large role: brine outflow vanishes periodically — e.g., when the atmospheric pressure drastically increases (Figure 5).

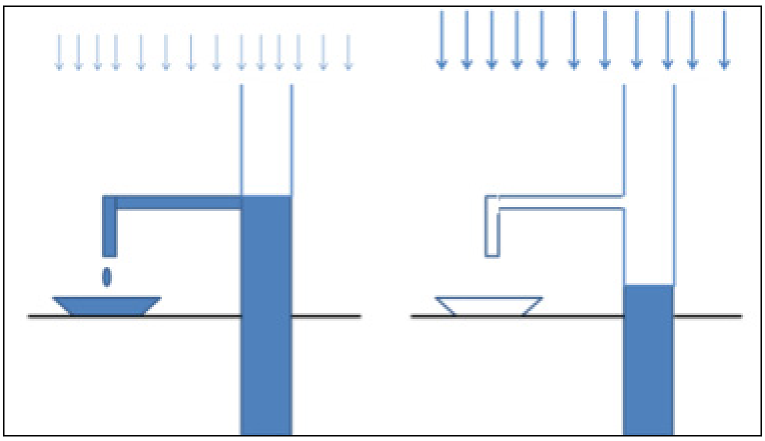

Figure 5 - Brine outflow from a shallow cavern: Left, low atmospheric pressure and brine flow from the cavern; right, rapidly increasing atmospheric pressure with no observed brine flow.

3. CAVERN COMPRESSIBILITY TEST

On July 3, 2009, cavern compressibility was measured by depressurizing SG13 from 0.18 MPa to zero. Brine was expelled from the cavern to a 500-litre container. An accurate flowmeter had been set at the SG13 wellhead, and cavern pressure was measured at the SG14 wellhead. (SG13 column composition changes during the test, because saturated brine rises in the well as a result of venting; for this reason, it was better to measure pressure evolution at the SG14 wellhead). Accurate *Paroscientific* pressure gauges were used. Figure 6 shows the expelled-brine-volume versus SG14-pressure-drop curve. The slope of this curve is the (as-measured) cavern compressibility, or $\beta V = 129.55\ \mathrm{m^3/MPa}$. When compared to cavern "sonar" volumes, this figure is relatively high. (In most caverns, the ratio between cavern compressibility and cavern volume is in the range $\beta \approx 4-5\times10^{-4}/\mathrm{MPa}$.)

Three hypotheses were considered:

(1) the cavern contains some gas;

(2) its flat shape makes the cavern more compressible; and

(3) the actual cavern volume is underestimated, as the sonar can "see" only the walls of the cavern that are reached by the sonar beams. This last assumption is supported by the value of the cumulated volume of injected water during cavern operation, which strongly suggests that the actual cavern volume might be as large as $V = 240{,}000\ \mathrm{m^3}$, a figure consistent with the as-measured cavern compressibility.

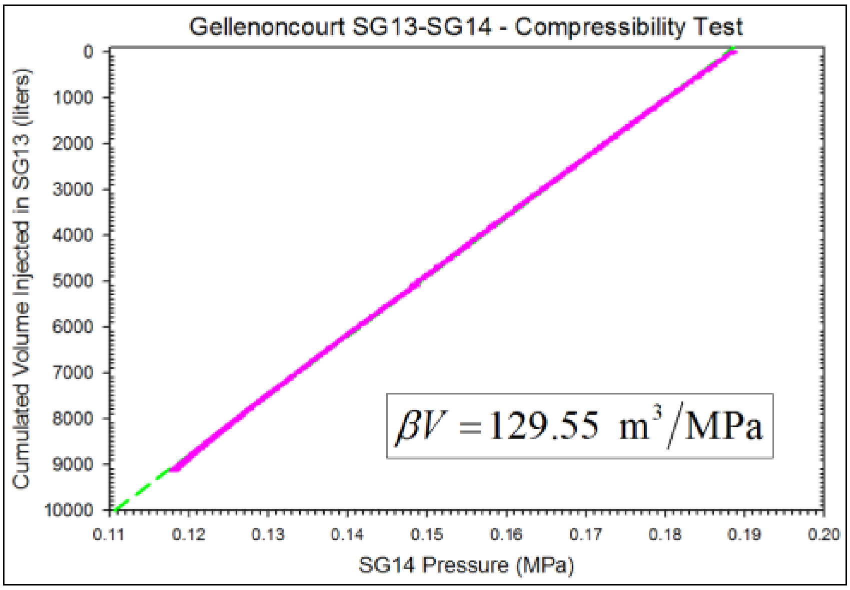

Figure 6 - Cavern compressibility measurement.

4. BRINE-OUTFLOW TEST

4.1 Outflow Measurement System

A general view of the outflow measurement system is given in Photo 1. A cabin was installed above the wellhead for security reasons. A solar panel was set on the cabin roof (Photo 1, right) to provide an energy supply. (The caverns are located far from the brine field station). A more detailed view of the brine measurement system is provided in Photo 2. Photo 1 (left) shows the upper part of the 7" casing. A hole was drilled through the steel tube to allow evacuation of the brine to a plastic container (Photo 3) whose weight is measured every minute. When this container is filled with brine, an electric valve automatically triggers container venting. A plastic cylinder was set above the string to prevent overflow. (After a rapid drop in atmospheric pressure, brine flow sometimes may be very fast, resulting in the rise of the air/brine interface above the hole for a couple of minutes, before overflowing brine is evacuated to the container through the hole.)

Photo 1 – Measurement system (left); energy supplied by a solar panel (right).

Photo 2 – Close-up of the measurement system.

Photo 3 – The hole above the 7" string that allows brine evacuation.

4.2 Average Brine-Flow-Rate

The test started on July 23 2008. The cumulated volume of expelled brine as a function of time is shown in Figure 7. Some uncertainty does exist; for instance, the brine outflow during container venting periods is not taken into account. The average brine-outflow rate (i.e., the overall amount of brine expelled during the testing period divided by the testing period duration) is $Q_{av} \approx 12$ litres/day .

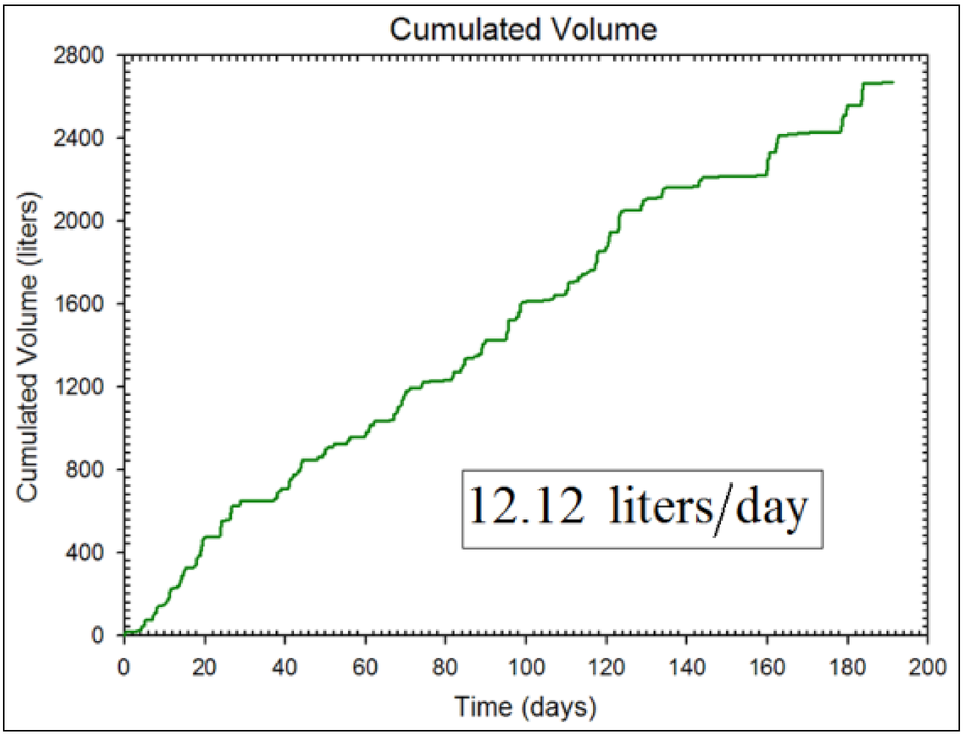

Figure 7 - Cumulated expelled volume as a function of time, or $v = v(t)$.

It will be seen, below, that this flow rate can be considered representative of the cavern-creep closure rate. When this flow is compared to cavern "sonar" volume, or $V \approx 135,000 \text{ m}^3$, the relative convergence rate is $\dot{V}/V = 1.1\times10^{-12} \text{ s}^{-1} = 3.65\times10^{-5} \text{ year}^{-1}$. However, when a more realistic value of cavern volume is considered, or $V = 240,000 \text{ m}^3$, the relative convergence rate is $\dot{V}/V = 5.8\times10^{-13} \text{ s}^{-1} = 1.8\times10^{-5} \text{ year}^{-1}$.

Even though slow, this rate is significantly faster than expected. At a 1000-m depth, the convergence rate typically is $\dot{V}/V = 3\times10^{-4} \text{ year}^{-1}$ [the figure observed during an outflow test performed in a 950-m deep cavern operated by GDF-Suez at Etrez (Brouard, 1998)]. The exponent of the Norton-Hoff power law typically is $n = 3$: it can be expected that, at 250-m depth, the cavern convergence rate may be slower by a factor larger than $(250/1000)^n = 1/64$.

4.3. Brine Flow-Rate Fluctuations

The *average* brine flow-rate was computed in Section 4.2. However, from Figure 8, it can be seen that brine flow-rate is far from being constant. In fact, large fluctuations can be observed: periodically, the brine flow-rate is several hundreds of liters per day — i.e., larger than the average flow rate by one or two orders of magnitude. However, for most of the time, the flow-rate is nil: no flow is expelled from the cavern, and the air/brine interface drops down into the well. This question is addressed in the following section.

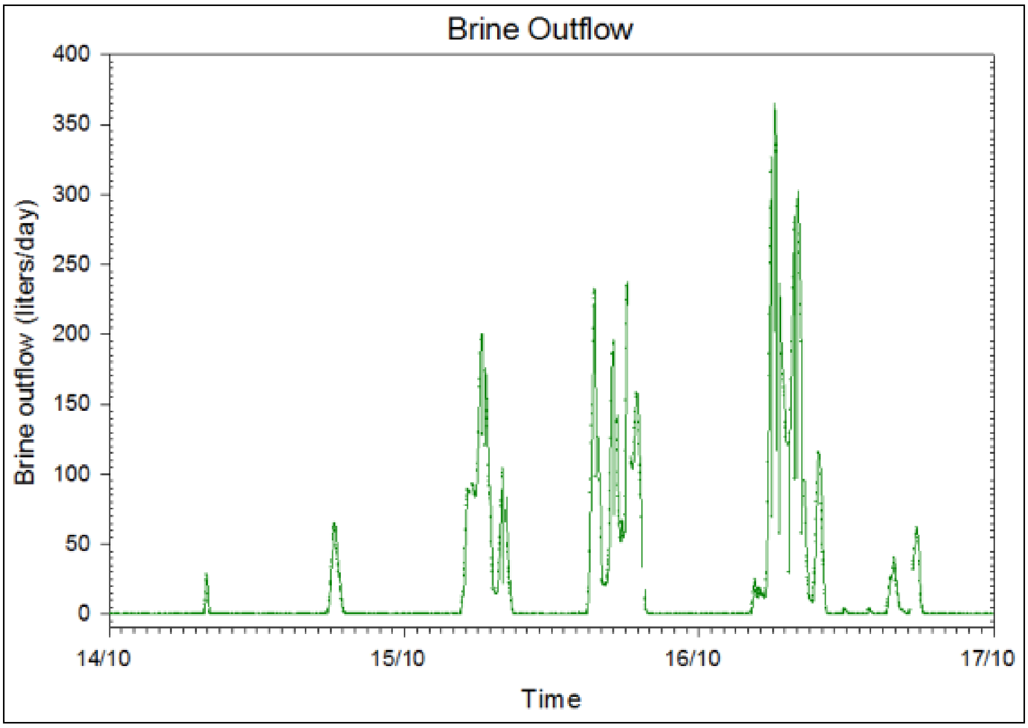

Figure 8 - Brine flow-rate as a function of time, flow rate computed every 10 minutes.

5. FLUCTUATIONS OF THE BRINE-OUTFLOW RATE

It has been said that the brine-outflow rate is influenced by cavern-creep closure, brine thermal expansion, brine permeation, brine leaks, atmospheric pressure variations, ground-level temperature variations and Earth tides. The effects of these factors are discussed in the following sections.

5.1. Brine Thermal Expansion

Brine thermal expansion is a real concern, as its effects often are large. Brine thermal expansion (or contraction) results from the gap between the temperature of the cavern brine and the geothermal temperature of the rock. When cavern brine is colder than the rock mass, heat is transferred from the rock mass to the cavern, resulting in brine warming. Conversely, when the brine is warmer than the rock mass, heat is transferred from the brine to the rock mass, resulting in brine cooling. Brine warming (or cooling) generates brine expansion (or contraction), which contributes to brine outflow. This process is slow — and even slower in a larger cavern. In a 240,000-m^3 cavern, it is expected that, after approximately 10 years, the initial temperature gap is divided by a factor of 4. For the SG13-SG14 cavern, soft water injected during the leaching process was slightly warmer (20°C) than the rock geothermal temperature, which typically is 16.6°C at cavern depth. The initial gap was small. Moreover, the cavern had been kept idle for nearly 30 years by the time after the brine-outflow test began. It is believed that thermal equilibrium nearly was reached at that time. However, by December 2008, a temperature gauge lowered into the SG13 well showed the cavern temperature to be perfectly constant. Temperature evolution is represented in Figure 9. The temperature apparently is perfectly constant during the period December 2008 – March 2009; however, this period is too short to allow for definite conclusions. (Although the temperature gauge resolution is 1/1000 °C, the accuracy of the temperature gauge is 1/100 °C; for the 4-month temperature measurement period, it can be inferred that temperature rate certainly is slower than $\dot{\theta} = 0.03°\ \text{C/yr}$. The brine expansion/contraction rate certainly is slower than $\alpha V \dot{\theta} = 10$ litres/day — possibly much slower. It is believed that a longer test period will prove that the actual temperature rate is exceedingly slow.)

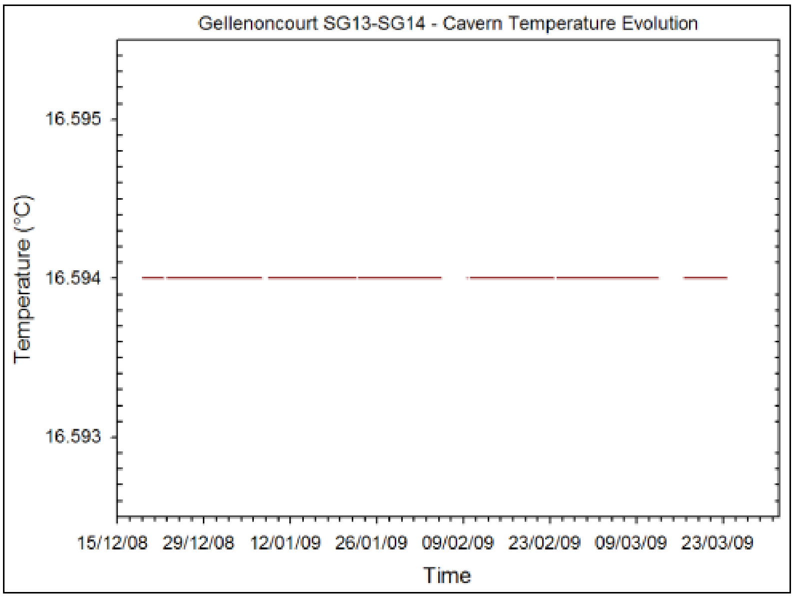

Figure 9 - Cavern temperature evolution from December 2008 to March 2009.

5.2. Brine Permeation and Brine Leaks

Because cavern brine pressure is halmostatic during the test, permeation and leaks are considered to be negligible. (The cavern pressure has remained halmostatic for 30 years; fast transient leak rates, sometimes observed at the beginning of a Mechanical Integrity Test, when a large pressure increase suddenly is applied in the cavern, were not expected during the SG13-SG14 test.)

4.4. Atmospheric Pressure Variation

Consider, first, atmospheric pressure variations (Figure 10). Cavern compressibility is approximately $\beta V = 130\ \mathrm{m}^3/\mathrm{MPa}$ (Figure 6) or, more conveniently when atmospheric pressure fluctuations are considered, $\beta V = 13$ liters/hPa. The magnitude of atmospheric pressure fluctuations is several hPa per day: one can expect that pressure fluctuations generate large changes in the brine-outflow rate, whose average value is 12 liters/day.

In fact, the ratio between the brine-outflow rate and atmospheric pressure fluctuations (or bV) is not exactly the same as the ratio between the injected brine flow and the cavern pressure increase, or $\beta V = 13$ liters/hPa. This is because, in sharp contrast to pressure changes during a compressibility test, atmospheric pressure applies an additional load both on the brine/air interface in the well, resulting in cavern expansion/contraction, *and* on the ground surface, resulting in a change in geostatic stresses and a contraction/expansion of the cavern (see the Appendix).

In an idealized spherical cavern, the "atmospheric" cavern compressibility is $bV = \beta_b V$, where $\beta_b \approx 2.7\times10^{-4}$/MPa is the saturated-brine compressibility factor. For the SG13-14 cavern, this rough estimation leads to $bV =$ 65 m^3/MPa.

The cumulated volume of brine expelled from the cavern is $v = v(t)$, and the atmospheric pressure evolution is $P_{atm} = P_{atm}(t)$. Figure 11 presents the "residual volume" — i.e., the cumulated volume of brine expelled during the 190-day test as a function of time *minus* the average volume of brine expelled during the test ($v(t) - Q_{av}t.$) *minus* the effects of atmospheric pressure fluctuations ($bV\left[P_{atm}(t) - P_{atm}(0)\right]$).

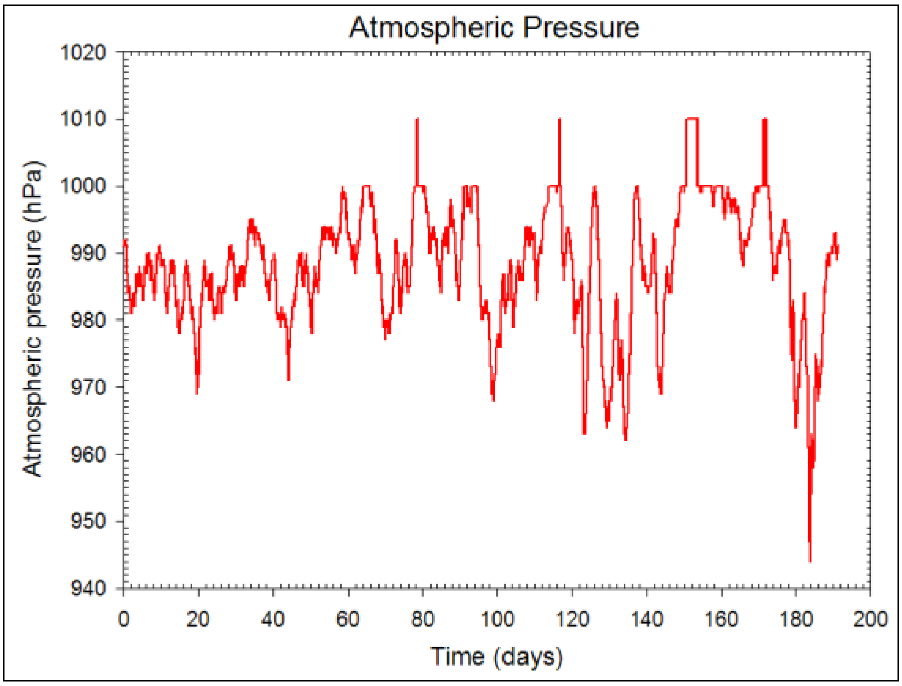

Figure 10 – Atmospheric pressure evolution during the test.

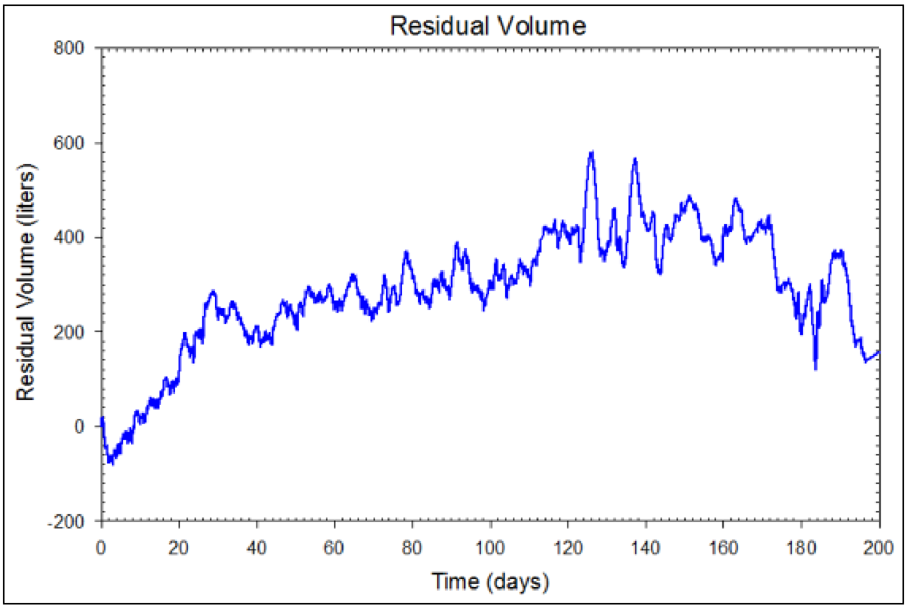

Figure 11 - "Residual" volume fluctuations.

4.5. Earth Tides and Ground Temperature Fluctuations

More-or-less periodic fluctuations are shown in Figure 11. It is known that ground-level temperature fluctuations, with a period of 24 hours, and Earth tides, with periods 12 hours -25 minutes and 24 hours, may have significant influence on cavern behavior (see, for example, Van Sambeek et al., 2005). Typically, Earth tides generate cavern volume changes that are in the range 10^{-8}-10^{-7}, or 2.4 to 24 liters in a $V = 240{,}000\ m^3$ cavern.

A Fourier analysis was performed to identify the main periods in the signal displayed in Figure 12.

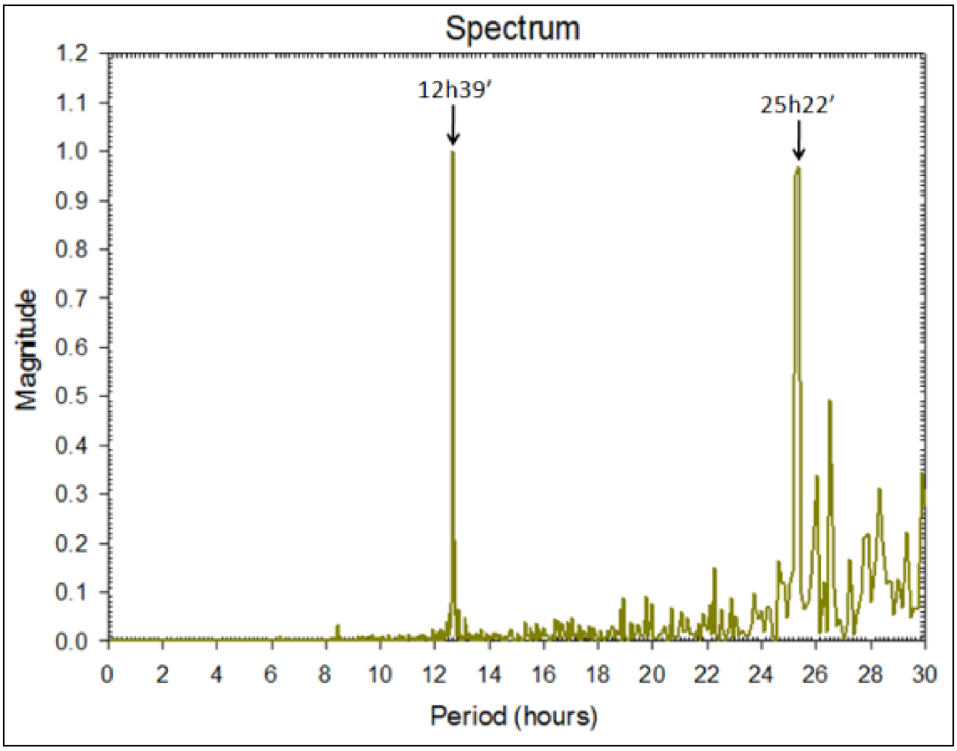

Figure 12 - Fourier analysis of the "residual" expelled volume.

Two periods clearly appear, 12 h 39’ and 25 h 22’, suggesting that, together with atmospheric pressure fluctuations, Earth tides play a significant role in the fluctuations of the observed rate of brine outflow during an outflow test in a shallow cavern.

CONCLUSIONS

A liquid-outflow test was performed in the SG13-14 cavern of the Gellenoncourt brine field, in advance of a cavern abandonment test. Brine outflow from the cavern was measured over a 190-day period. Brine outflow is a geyser-like phenomenon: fast brine rates (several hundreds of liters/day) are followed by long periods of time during which no flow is expelled from the cavern.

However, the average brine flow-rate is representative of the cavern-creep closure rate, which is $\dot{V}/V = 5.8\times10^{-13}\ \text{s}^{-1} = 1.8\times10^{-5}\ \text{year}^{-1}$. Although small, this closure rate is faster than expected.

Fluctuations of the brine flow rate can be explained to a large extent by the fluctuations of atmospheric pressure; however, “atmospheric” cavern compressibility is significantly smaller than standard cavern compressibility, and Earth tides also play a significant role.

ACKOWLEDGEMNTS

The authors are indebted to the staff of the Gellenoncourt CSME brine field, whose help in performing the tests was invaluable.

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APPENDIX — EFFECT OF ATMOSPHERIC PRESSURE

A1 - Relations between Atmospheric Pressure Variations, Cavern Pressure and Brine Outflow

Atmospheric pressure fluctuations are transmitted to the rock mass through the ground and to the cavern through the well, at least when the wellhead is left open, resulting in a cavern volume variation $bV_c\dot{P}_{atm}$.

One must distinguish between variations of the geostatic state of stress, which are proportional to atmospheric pressure variations $\dot{P}_{atm}$, and variations of the cavern brine pressure, or $\dot{P}_c$, which are linked to atmospheric pressure variations.

It often is assumed that variations of the geostatic state of stress in the rock mass (assuming no underground cavity) are connected to atmospheric pressure variations through the following equations:

$$\begin{cases} \dot{\sigma}_{zz} = -\dot{P}_{atm} \\ \dot{\sigma}_{xx} = \dot{\sigma}_{yy} = -\left[\nu/(1-\nu)\right]\dot{P}_{atm} \end{cases} \quad (1)$$

where ν is the Poisson's ratio of the rock mass. This so-called "oedometric" hypothesis usually is considered when effects of atmospheric pressure on aquifer layers are studied.

Considering an open cavern, variations of atmospheric pressure that are transmitted to the cavity through the rock mass generate a cavern-volume variation $\dot{V}_c^1$:

$$\dot{V}_c^1 = -\beta_\infty V_c \times \dot{P}_{atm} \quad (2)$$

An increase of cavern pressure, $\dot{P}_c$, triggers a cavern-volume variation $\dot{V}_c^2$:

$$\dot{V}_c^2 = -\beta_c V_c \times \dot{P}_c \quad (3)$$

Therefore

$$\dot{V}_c = \dot{V}_c^1 + \dot{V}_c^2 \quad (4)$$

A variation of cavern pressure, $\dot{P}_c$, also triggers a brine-volume variation $\dot{V}_b$:

$$\dot{V}_b = -\beta_b V_b \times \dot{P}_c \approx -\beta_b V_c \times \dot{P}_c \quad (5)$$

Then, the following three cases can be distinguished.

Case 1. When the wellhead is closed and the well fully is filled with brine, then $\dot{V}_b = \dot{V}_c^1 + \dot{V}_c^2$, and no other mechanism leads to pressure increase. Variations of atmospheric pressure are transmitted to the cavern only though the rock mass:

$$\boxed{\dot{P}_c = \frac{\beta_\infty}{\beta_b + \beta_c}\dot{P}_{atm}} \quad (6)$$

Case 2. When the wellhead is left open, no other mechanism leads to pressure increase, and atmospheric pressure decreases. Brine outflow is $Q = \dot{V}_b - \dot{V}_c^1 - \dot{V}_c^2$, and atmospheric pressure variations are transmitted to the cavern both though the rock mass and the well:

$$Q = (\beta_b + \beta_c - \beta_\infty)V \times \dot{P}_{atm} \quad (7)$$

Case 3. When the wellhead is left open, no other mechanism leads to pressure increase, and atmospheric pressure rapidly increases. The air/brine interface drops down into the well, $h > 0$, and the brine inflow in the well ($Q > 0$ if interface rises, $Q < 0$ is interface lowers) is $Q = -S\dot{h} = \dot{V}_b - \dot{V}_c^1 - \dot{V}_c^2$ and $\dot{P}_c = \dot{P}_{atm} - \rho_b g\dot{h}$:

$$Q = (\beta_b + \beta_c - \beta_\infty)V \times \dot{P}_{atm} = -\left[S + (\beta_b + \beta_c)V\rho_b g\right]\dot{h} \quad (8)$$

where h is air/brine interface depth, S is inner cross-section of the well, and ρ_b is saturated brine density.

A2 - The Case of a Spherical Cavern

The two coefficients β_c and β_∞, defined above, can be calculated using a finite-element codes for any given shape of the cavern when elastic properties of the salt mass and overburden layers are known. Considering the simple case of a spherical cavern, radius R, in a semi-infinite medium, the Lamé closed-formed solution can be used. If the cavern pressure variation is $\dot{P}_c$ and if the geostatic pressure variation — in all directions — far from the cavern is $\dot{P}_\infty$, then stress variations in the rock mass are as follows:

$$\begin{cases} \dot{\sigma}_{rr}(r) = -\dot{P}_\infty + (\dot{P}_\infty - \dot{P}_c)\dfrac{R^3}{r^3} \\ \dot{\sigma}_{\theta\theta}(r) = -\dot{P}_a - \dfrac{1}{2}(\dot{P}_\infty - \dot{P}_c)\dfrac{R^3}{r^3} \\ E\dfrac{u}{r} = -(1-2\nu)\dot{P}_\infty - \dfrac{(1+\nu)}{2}(\dot{P}_\infty - \dot{P}_c)\dfrac{R^3}{r^3} \end{cases} \quad (9)$$

- First, considering only cavern pressure variation ($\dot{P}_\infty = 0$), the cavern-volume variation can be written as:

$$\frac{\dot{V}_c}{V} = \frac{3}{2}(1+\nu)\frac{\dot{P}_c}{E} \qquad \text{where} \qquad \boxed{\beta_c = \frac{3}{2}\frac{(1+\nu)}{E}} \quad (10)$$

- Considering here the case when $\dot{P}_c = 0$, the cavern-volume variation can be written as:

$$\frac{\dot{V}_c}{V_c} = -\frac{9}{2}(1-\nu)\frac{\dot{P}_{atm}}{E} \quad (11)$$

Further calculations lead to the following formula:

$$\boxed{\beta_\infty = \frac{3}{2}\frac{(1+\nu)}{E}} \quad (12)$$

Therefore, in Case 2 (open cavern and decreasing atmospheric pressure), $\dot{P}_c = \dot{P}_{atm}$ and $\beta_\infty = \beta_c$. This means that for the brine outflow of a spherical cavern, Q is directly proportional to atmospheric pressure variations, or $\beta \approx b$:

$$\boxed{Q = \beta_b V \times \dot{P}_{atm}} \quad (13)$$

In other words the “atmospheric” cavern compressibility, or the ratio between the outflow rate and the atmospheric pressure rate, is smaller than the standard cavern compressibility — i.e., the ratio between the rate of liquid injection and the pressure increase rate when the cavern is closed.